Methodology of Drought Risk assessment in DriDanube project

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Outline

1 Introduction
2 Theoretical, mathematical background
3 Algorithm for drought risk assessment
4 Software RED for drought risk assessment
5 Risk Maps
6 Software Practice (tomorrow)
Introduction

One of the aim of the DriDanube project is the drought risk assessment and the main responsible partner of this activity is the Hungarian Meteorological Service.

However, unfortunately there is no commonly accepted procedure for this purpose therefore it was necessary to review and prepare the mathematical methodology that can be applied for drought risk assessment in this project. VARIMAX is a subcontractor for the mathematical and software development.

The recommended approach was as it is described in European Commission’s Risk Assessment and Mapping Guidelines for Disaster Management.
5. Risk Assessment Methods
5.1. Conceptual Framework and Basic Methodology

According to ISO 31010, risks are the combination of the consequences of an event or hazard and the associated likelihood of its occurrence.

Risk = hazard impact * probability of occurrence

Qualitative Formulation only!
What is the Risk Matrix? **It is a tool for Qualitative Risk Mapping!**

Schema of Qualitative Risk Mapping according to the Methodology of SEERISK

![Diagram of Qualitative Risk Mapping](image)

The two qualitative maps (probability and impact) can be overlaid. Each pixel will be given a colour which will indicate a risk level according to the risk matrix.

Qualitative data will lead to the production of a risk matrix that will then be used as a base for the production of a qualitative risk map.

**But we want to implement Quantitative Risk Mapping!**
General Mathematical Methodology
(for meteorological drought risk assessment)

Meteorological variables in space (s) and time (t year):

\[ X(s,t) = [X_1(s,t), \ldots, X_N(s,t)]^T \quad s \in S \quad \text{(e.g. grid)}, \quad t = 1, \ldots, n \]

i, Continuous distribution, joint density function: \( f(x; s) \)

ii, Discrete distribution: \( p_i(s) = P(X(s,t) = a_i) \) , \( i = 1, 2, \ldots \)
Meteorological drought loss (hazard impact): \( \text{Loss}(s,t) \)

Meteorological drought loss (hazard impact) function:

\[
E\left( \text{Loss}(s,t) \mid \textbf{X}(s,t) \right) = \text{L}(\textbf{X}(s,t))
\]

\( \text{L}(\textbf{X}(s,t)) \) that is the conditional expectation (regression) of the meteorological loss given the meteorological variables.

Meteorological drought risk (depends on location):

\[
Risk(s) = E\left( \text{Loss}(s,t) \right) = E\left( \text{L}(\textbf{X}(s,t)) \right)
\]

i.e. the expected value of the meteorological loss function.
The basic cases

i, Continuous distribution: \( Risk (s) = E \left( L(X(s,t)) \right) = \int_{\mathbb{R}^N} L(x) f(x; s) \, dx \)

ii, Discrete distribution: \( Risk (s) = E \left( L(X(s,t)) \right) = \sum_{i=1}^{\infty} L(a_i) \cdot p_i(s) \)

Special case:
If \( L(a_1) = \text{hazard impact} \), \( p_1(s) = \text{probability of occurrence} \)
\( L(a_2) = 0 \), \( p_2(s) = 1 - p_1(s) \), then
\( Risk (s) = \sum_{i=1}^{2} L(a_i) \cdot p_i(s) = \text{hazard impact} \times \text{probability of occurrence} \)
Estimation of meteorological drought risk

\[ \hat{Risk}(s) = \frac{1}{n} \sum_{t=1}^{n} \hat{L}(X(s,t)) \]

where \( \hat{L}(X(s,t)) \) is an estimation of loss function \( L(X(s,t)) \),

since \( Risk(s) = E\left( L(X(s,t)) \right) \)

BUT: In general there is no sample for the drought loss values!
Loss function derivation from yield function and drought identification

Crop yield values: \( Y(s,t) = \text{Yield}(s,t) \)

Meteorological yield function:

\[
E \left( \text{Yield}(s,t) \mid X(s,t) \right) = Y(X(s,t))
\]

that is the conditional expectation (regression) of the crop yield given the meteorological variables.

**Drought identification by set**  \( D \): there is drought if \( X(s,t) \in D \)

**The probability of drought:** \( P_D = P(X(s,t) \in D) \)
Loss function based on yield function and drought identification

\[ L(X(s, t)) = 0 \quad \text{if} \quad X(s, t) \not\in D \quad \text{(there is no drought)} \]

\[ L(X(s, t)) = \mathbb{E}(Y(X(s, t)) \mid X(s, t) \not\in D) - Y(X(s, t)) \quad \text{if} \quad X(s, t) \in D \quad \text{(there is drought)} \]

\[ \mathbb{E}(Y(X(s, t)) \mid X(s, t) \not\in D) : \text{conditional expectation of yield if there is no drought} \]

Then risk can be expressed as,

\[ Risk = \mathbb{E}(L(X)) = \left( \mathbb{E}(Y(X) \mid X \not\in D) - \mathbb{E}(Y(X) \mid X \in D) \right) \cdot P_D \]
Special case: definition of set $D$ by an SPI

$$D = \{ x \mid SPI < C_p \} \text{ where } \Pr(SPI < C_p) = \Phi(C_p) = P_D$$

and $\Phi(x)$ is the standard normal distribution function.

In this project SPI was used for drought identification.

However other optional drought index or drought definition can be used!
Estimation of loss function based on estimated yield function and SPI

Let $\hat{Y}(X(s, t)) (t = 1,..,n)$ be an estimation of the yield function based on a sample.

Estimation of loss function using estimated yield function and SPI:

$\hat{L}(X(s, t)) = 0$ if $SPI(s, t) \geq C_p$ (there is no drought)

$\hat{L}(X(s, t)) = \left( \frac{1}{n - n_D} \sum_{SPI(s,t) \geq C_p} \hat{Y}(X(s, t)) \right) - \hat{Y}(X(s, t))$ if $SPI(s, t) < C_p$ (there is drought)

$n_D = \sum_{SPI(s,t) < C_p} 1$ : frequency of drought
Estimation of drought risk based on estimated yield function and SPI

\[ \hat{\text{Risk}}(s) = \frac{1}{n} \sum_{t=1}^{n} \hat{L}(X(s,t)) = \left( \frac{1}{n-n_D} \sum_{\text{SPI}(s,t) \geq C_P} \hat{Y}(X(s,t)) \right) - \left( \frac{1}{n_D} \sum_{\text{SPI}(s,t) < C_P} \hat{Y}(X(s,t)) \right) \cdot \hat{P}_D \]

where \( \hat{P}_D = \frac{n_D}{n} \) is the estimated probability of drought.

It is an aesthetic formula, thus it must be good!
Remark about the nature of Risk

The structure of Drought Risk: $\text{Risk}_D = \mathbb{E}(L_D) = \mathbb{E}(L_D | D) \cdot P_D$

$D$: drought  $L_D$: loss  $P_D$: probability of drought

Different definitions of drought: $D_1, D_2$

Maybe: $P_{D_1} < P_{D_2}$ but $\mathbb{E}(L_{D_1} | D_1) > \mathbb{E}(L_{D_2} | D_2)$ and $\text{Risk}_{D_1} = \mathbb{E}(L_{D_1}) \approx \mathbb{E}(L_{D_2}) = \text{Risk}_{D_2}$

Similarly to the Roulette

If we have 1 jeton then for all the bets: $\text{Risk} = \mathbb{E}(\text{Loss}) = \frac{1}{37}$

Single number bet: $\text{Risk} = \mathbb{E}(\text{Loss}) = 1 \cdot \frac{36}{37} - 35 \cdot \frac{1}{37} = \frac{1}{37}$

Red or black bet: $\text{Risk} = \mathbb{E}(\text{Loss}) = 1 \cdot \frac{19}{37} - 1 \cdot \frac{18}{37} = \frac{1}{37}$
Estimation of the relative yield function that was used

Relative yield in percent: \( Y_{rel}(s,t) = \frac{Y(s,t) - E(Y(s,t))}{E(Y(s,t))} \cdot 100\% \)

Using the equality \( E(Y_{rel}(s,t)) \equiv 0 \) the following regression model was applied,

\[
\hat{Y}_{rel}(X(s,t)) = \alpha_1 \cdot PI(s,t) + \alpha_2 \cdot TI(s,t)
\]

using prec. index \( PI(s,t) = \ln X_1(s,t) - E_{ln,1}(s) \) and temp. index \( TI(s,t) = X_2(s,t) - E_2(s) \),

where \( X_1(s,t) \) is precipitation sum and \( X_2(s,t) \) is temperature mean for a given 3 or 6 months period, and \( E_{ln,1}(s), E_2(s) \) are the spatial expected values.

The common coefficients \( \alpha_1, \alpha_2 \) can be estimated by the method of least squares.

At the risk estimation the \( SPI(X_1(s,t)) \) can be applied. Then we can evaluate the connection of the different period SPI series with the real drought loss.

The regression coefficients \( \alpha_1, \alpha_2 \) were estimated for several 3 or 6 months periods and four plants: maize, wheat, rape and barley.
Algorithm for drought risk assessment applied in the project

1. Selection of the relevant meteorological variables $X(s, t)$. Monthly precipitation and temperature series can be selected.

2. Collecting sample for the crop yield $Yield(s, t)$ and the meteorological variables $X(s, t)$.

3. Estimation of the relative crop yield function $\hat{Y}_{rel}(X(s, t))$ that is a regression of the relative crop yield on the meteorological variables.

4. On the basis of the results of item 3 developing software for calculation of the following series and estimations in case of given meteorological data series $X(s, t) (t = 1,..,n)$:
   - Estimated relative crop yield functions $\hat{Y}_{rel}(X(s, t)) (t = 1,..,n)$ according to item 3.
   - Several $SPI(s, t) (t = 1,..,n)$ series.
   - Estimated meteorological drought risk $\hat{Risk}(s)$.


6. Mapping of risk outside CarpatClim, DanubeClim area based on station data series and possibly other gridded data series (e.g. E-Obs).
Risk Estimation of Drought  
(RED v1.01)  
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I. INTRODUCTION

II. MATHEMATICAL BACKGROUND

III. THE STRUCTURE OF PROGRAM SYSTEM

IV. THE MAIN STEPS OF RISK ESTIMATION

V. THE MAIN INPUT/OUTPUT FILES

VI. REGRESSION PARAMETER FILES (REG.PAR)

VII. EXAMPLE: MAPPING OF RISK FOR HUNGARY
Some Explanation

I retired from the Hungarian Meteorological Service.

I continue my activity in VARIMAX Limited Partnership.

Software RED: Risk Estimation of Drought

Movie RED: Retired, Extremely Dangerous
1. INDEX CALCULATION

SPI calculation, selection
(1.1.1-2)

PI calculation, selection
(1.2.1-2)

TI calculation, selection
(1.3.1-2)

Recalibration of SPI, PI, TI
(1.4)

2. RISK CALCULATION

Regression of relative crop yield
(2.1)

Definition of drought by SPI
(2.2)

Definition of drought by other input
(Remark 1)

Estimation of Risk
(2.3)

Estimation of Risk
(2.4)

Possible other input
crop yield and drought data
(Remark 2)
CarpatClim and DanubeClim database

Daily gridded data series for basic meteorological variables (1961-2010)
Spatial resolution: 0.1°
Methodology: MASH for homogenization, MISH for gridding
Project of JRC (2011-2013) (10 participants)

Same methods: MISH-MASH
Bilateral contracts with JRC
Based on gridded data series

Spatial resolution 0.1° (6913 grid)

Output risk.res values presented

Risk Map for CarpatClim and DanubeClim area

Risk Map for Maize (period: 5-7, drought probability: 0.2)
There is no royal road! (Archimedes)

Thank you for your attention